Transmission Loss Estimation of Three Dimensional Silencers with Perforated Internal Structures Using Multi-domain BEM

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The calculation of the transmission loss of the silencers with complicated internal structures by the conventional BEM combined with the transfer matrix method is incorrect at best or impossible for 3-dimensional silencers due to its inherent plane wave assumption. On this consideration, we propose an efficient practical means to formulate algebraic overall condensed acoustic equations for the whole acoustic structure, where particle velocities on the domain interface boundaries are unknowns, and the solutions are used later to compute the overall transfer matrix elements, based on the multi-domain BEM data. The transmission loss estimation by the proposed method is tested by comparison with the experimental one on an air suction silencer with perforated internal structures installed in air compressors. The method shows its viability by presenting the reasonably consistent anticipation of the experimental result.

Key Words: Transmission Loss, Overall Condensed Acoustic Equation, Multi-domain BEM (Boundary Element Method), Air Suction Silencer

1. Introduction

In general, only simple shaped silencers may be analyzed by conventional analytical approach. The silencers of complicated shape with or without acoustic internal structures have been analyzed by transfer matrix method (Munjal, 1987), FEM (Finite Element Method), the transfer matrix method with BEM (Tanaka and Fujikawa, 1985), Multi-domain BEM (Boundary Element Method) (Cheng and Seybert, 1991), Multi-Domain Structural-Acoustic Coupling Analysis (Ju and Lee, 2001), etc. Among these, the transfer matrix method with BEM has been used to calculate the transmission or insertion loss of the silencers. In this method, the particle velocities and sound pressures of each domain of the silencer are calculated by BEM and the four pole parameters of every domain are calculated. The overall transfer matrix of the silencer is calculated through serial multiplication of the domain transfer matrices. The transfer matrices are constructed under the assumption of plane wave propagations which is only true below the cutoff frequency, and should be applied with care. When the silencers with complicated 3-dimensional structures, e.g., ones with perforated internal structures are divided into several domains for the BEM analysis, plane wave propagations in the inlet and outlet boundaries of each domain can't be assumed anymore and above method can not be applied.

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A direct mixed-body BEM (Wu et al., 1998) to model mufflers with perforated plates was proposed but the method produces a hyper-singular integral and too many variables in the domains.

To cope with this problem, we develop a transmission loss estimation program for 3-dimensional silencers using multi-domain BEM which considerably reduces the computational burden with no sacrifice of the accuracy. The transmission loss with perforated internal structures or thin obstacles or internal slender pipe configuration inside the silencer can be solved by multi-domain BEM with the impedance of perforated interface structures which can be measured experimentally. Treatment of the perforated internal structures as impedance components puts a way to evade the interrupting assumption of the pressure and particle velocity continuity on the domain boundaries in the previous study (Cheng and Seybert, 1991). With this concept, we formulate linear algebraic overall acoustic equations for the whole acoustic structure, condensed only with unknown particle velocities, excluding the pressure unknowns, on the domain interface boundaries, the solutions of which are used later to compute the overall transfer matrix elements, based on the multi-domain BEM data. The coefficient matrix of the overall equations are so sparse and blocked that some systematic solution procedure can be adopted. The transfer matrix elements are obtained by solving the acoustic variables at the inlet and outlet domains of the silencer. This calculation process may be applied effectively to the acoustic analysis and design of three-dimensional silencers with "cascaded" sub-domains that contain perforated internal structures, thin obstacles or internal slender pipe configuration, and other complicated 3-dimensional acoustic structures.

The accuracy of the developed program is validated by experimental work through two application trials for the transmission loss determination in a simple expansion silencer and an air suction silencer of an air compressor. The simple expansion chamber has been chosen for a simple evaluation with its exact analytical solution within the plane wave surviving frequency range. Air compressors are used as important utility equipments in so many various fields, but known as notorious noisy sources in the installing facilities (Jeon et al., 2004; Ju et al., 2004). In many cases, the inlet air filters in the compressors are charged as the noise silencing role simultaneously. That is the reason why the inlet air filter of the compressor is considered as the air suction silencer in our paper. The perforated internal filter-guide structures and the short inlet duct, expansion chamber, and outlet duct make the silencer structure acoustically complicated. Therefore, we select this as a suitable validating example for our suggesting method and program.

2. Multi-Domain BEM

Acoustic cavities enclosed by a surface structure can be divided into several domains to yield a multi-domain problem. The boundary integral formulation for the problem can be explained by considering a three dimensional enclosed structure with multi-domains, Ω_{I}, Ω_{II}, $\cdots$, Ω_i, $\cdots$, Ω_N as shown in Fig. 1.

First, it is assumed that the cavity volume Ω surrounded by a boundary S, is splitted into two partial domains Ω_{I} and Ω_{II}. The domain Ω_{I} is enclosed by the intrinsical boundary S_1^{I} and the new interface boundary S_1^{II}. Similarly, the intrinsical boundary S_2^{II} and the boundary S_1^{II} interfacing Ω_{I} and Ω_{II} encapsulates the domain Ω_{II}. In dividing domains, the splitting interface wall between the two domains behaves as a common partial boundary for the splitted domains.

The acoustic system of equations for the partial domain, Ω_{I} can be formulated as follows. The fluid in Ω_{I} is treated as compressible, inviscid, and non-flowing fluid medium. For time-harmonic excitation, the velocity potential ϕ in the fluid must satisfy the Kirchhoff-Helmholtz integral

Fig. 1 Divided domains of the acoustic structure

equation.

$$C^{o}(p)\phi(p) = \int_S \left\{ \psi(P, Q) \frac{\partial \phi}{\partial n}(Q) - \phi(Q) \frac{\partial \psi}{\partial n}(P, Q) \right\} dS(Q) \quad (1a)$$

where P is a collocation point, Q is any point on S and n denotes the coordinate normal to the surface. The function ψ is the three-dimensional free-domain Green's function, $\psi(P, Q) = \mathrm{esp}[-ikR(P, Q)]/R(P, Q)$, in which $R(P, Q)$ is the distance between P and Q. The coefficient $C^{o}(P)$ has the value 4π for P in Ω_{I} and on any arbitrary surface can be evaluated by the following equation (Seybert et al., 1985).

$$C^{o}(p) = -\int_S \frac{\partial}{\partial n}\left(\frac{1}{R(P, Q)}\right) dS(Q) \quad (1b)$$

By discretization, Eq. (1a) can be transformed into algebraic equations for the domain one, Ω_{I} as follows :

$$\sum_{l=1}^{M} B_{jl}^{\mathrm{I}} \cdot p_l^{\mathrm{I}} = \sum_{l=1}^{M} A_{jl}^{\mathrm{I}} \cdot u_l^{\mathrm{I}} \quad (j=1, 2, \cdots, M) \quad (2a)$$

or

$$[B^{\mathrm{I}}]\{p^{\mathrm{I}}\} = [A^{\mathrm{I}}]\{u^{\mathrm{I}}\} \quad (2b)$$

where M represents the number of collocation points (the number of nodes on the boundary surface), l denotes the l-th collocation point, j denotes the j-th node on the boundary, the superscript I indicates the partial domain I, and $\{p^{\mathrm{I}}\}$ and $\{u^{\mathrm{I}}\}$ represent sound pressures and particle velocities, respectively.

Similarly, for the other domain II, Ω_{II}, we formulate the following equivalence.

$$[B^{\mathrm{II}}]\{p^{\mathrm{II}}\} = [A^{\mathrm{II}}]\{u^{\mathrm{II}}\} \quad (3)$$

For example, consider a case in which $\{u_1^{\mathrm{I}}\}$ and $\{u_2^{\mathrm{II}}\}$ are known and $\{p_1^{\mathrm{II}}\}$ and $\{u_1^{\mathrm{II}}\}$ are unknown but acoustic impedance is given on the boundary S_1^{II}.

Here, we rewrite Eq. (2b) as

$$\begin{bmatrix} \{p_1^{\mathrm{I}}\} \\ \{p_2^{\mathrm{I}}\} \end{bmatrix} = \begin{bmatrix} [D_{11}^{\mathrm{I}}] & [D_{12}^{\mathrm{I}}] \\ [D_{21}^{\mathrm{I}}] & [D_{22}^{\mathrm{I}}] \end{bmatrix} \begin{bmatrix} \{u_1^{\mathrm{I}}\} \\ \{u_2^{\mathrm{I}}\} \end{bmatrix} \quad (4)$$

where

$$\begin{bmatrix} [D_{11}^{\mathrm{I}}] & [D_{12}^{\mathrm{I}}] \\ [D_{21}^{\mathrm{I}}] & [D_{22}^{\mathrm{I}}] \end{bmatrix} = \begin{bmatrix} [B_{11}^{\mathrm{I}}] & [B_{12}^{\mathrm{I}}] \\ [B_{21}^{\mathrm{I}}] & [B_{22}^{\mathrm{I}}] \end{bmatrix}^{-1} \cdot \begin{bmatrix} [A_{11}^{\mathrm{I}}] & [A_{12}^{\mathrm{I}}] \\ [A_{21}^{\mathrm{I}}] & [A_{22}^{\mathrm{I}}] \end{bmatrix} \quad (5)$$

and $[B^{\mathrm{I}}]^{-1}$ means the inverse matrix of $[B^{\mathrm{I}}]$.

The particle velocities of both domains on the interface boundary should satisfy Eq. (6).

$$\{u_1^{\mathrm{II}}\} = -\{u_2^{\mathrm{I}}\} \quad (6)$$

Then, the sound pressures $\{p_2^{\mathrm{I}}\}$ can be expressed from the second row of Eq. (4) and Eq. (6) as

$$\{p_2^{\mathrm{I}}\} = [D_{21}^{\mathrm{I}}]\{u_1^{\mathrm{I}}\} - [D_{22}^{\mathrm{I}}]\{u_1^{\mathrm{II}}\} \quad (7)$$

Similarly, the sound pressures $\{p_1^{\mathrm{II}}\}$ depend on the particle velocities $\{u_1^{\mathrm{II}}\}$ and $\{u_2^{\mathrm{II}}\}$ on the boundaries of Ω_{II} in the fashion of

$$\{p_1^{\mathrm{II}}\} = [D_{11}^{\mathrm{II}}]\{u_1^{\mathrm{II}}\} + [D_{12}^{\mathrm{II}}]\{u_2^{\mathrm{II}}\} \quad (8)$$

If the impedance of the interface boundary between the two domains is given, the sound pressures and particle velocities on the interface can be related as follows :

$$\{p_1^{\mathrm{II}}\} - \{p_2^{\mathrm{I}}\} = [Z_{\mathrm{I\,II}}]\{u_1^{\mathrm{II}}\} \quad (9)$$

where $Z_{\mathrm{I\,II}}$ is the impedance matrix of the interface boundary.

Eq. (9) substituted with Eq. (7) and Eq. (8) can be rewritten for the interface particle velocities of the second domain, $\{u_1^{\mathrm{II}}\}$ as follows :

$$\{u_1^{\mathrm{II}}\} = [[Z_{\mathrm{I\,II}}] - [D_{11}^{\mathrm{II}}] - [D_{22}^{\mathrm{I}}]]^{-1} \cdot [[D_{12}^{\mathrm{II}}]\{u_2^{\mathrm{II}}\} - [D_{21}^{\mathrm{I}}]\{u_1^{\mathrm{I}}\}] \quad (10)$$

The sound pressures $\{p_1^{\mathrm{I}}\}$ can be found from the first row of Eq. (4) and Eq. (6) as

$$\{p_1^{\mathrm{I}}\} = [D_{11}^{\mathrm{I}}]\{u_1^{\mathrm{I}}\} - [D_{12}^{\mathrm{I}}]\{u_1^{\mathrm{II}}\} \quad (11)$$

Finally, the unknown $\{p_1^{\mathrm{I}}\}$ can be solved by replacing the known $\{u_1^{\mathrm{I}}\}$ and $\{u_1^{\mathrm{II}}\}$.

3. Transmission Loss of the 3-dimensional Silencer with Internal Perforated Structures

Transmission loss is defined by the ratio of the inlet and outlet sound powers of the interested acoustic structure. The transmission loss can be expressed as follows (Wu et al., 1998):

$$TL=20\log_{10}\left\{\frac{1}{2}\left|T_{11}+\frac{T_{12}}{z_o}+T_{21}z_o+T_{22}\right|\right\} +10\log_{10}S_i/S_o \tag{12}$$

where z_o is the characteristic impedance, S_i and S_o are the cross-sectional areas of the inlet and outlet tubes, and T_{11}, T_{12}, T_{21}, and are the four-pole parameters between the inlet and outlet of the acoustic structure.

Here, we present the procedure to determine the transmission loss of the air suction silencer shown in Fig. 2. On considering the acoustic structural characteristics of the air suction silencer, we divide the three dimensional enclosed structure into four domains, Ω_{I}, Ω_{II}, Ω_{III} and Ω_{IV} as shown in Fig. 3. The domains Ω_{I} and Ω_{IV} are the inlet and outlet duct volume, respectively. And Ω_{II} and Ω_{III} represent the inner and outer volume of the main room splitted by the air filter inserted in the annular space formed by two perforated plates, respectively. Calculation process of the intermediate acoustic variables for four-pole parameters of the silencer is detailed in Appendix.

Fig. 2 Boundary element model of the air suction silencer

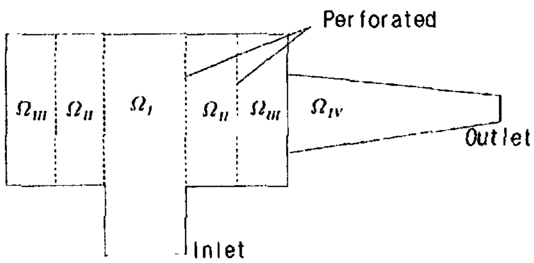

Fig. 3 Four domains of the air suction silencer with internal perforated structures

First, to obtain T_{11} and T_{21}, the unknown particle velocity equation for the overall acoustic system formed by the four domains, Ω_{I}, Ω_{II}, Ω_{III} and Ω_{IV}, is formulated by reassembling the intermediate equations obtained using the multi-domain BEM data and presented in Appendix A.1, as follows:

$$\begin{pmatrix} [D_{22}^{\mathrm{I}}+D_{11}^{\mathrm{II}}-Z_{\mathrm{I\,II}}] & [-D_{13}^{\mathrm{II}}] & [0] \\ [-D_{31}^{\mathrm{II}}] & [D_{11}^{\mathrm{III}}+D_{33}^{\mathrm{II}}-Z_{\mathrm{II\,III}}] & [-D_{13}^{\mathrm{III}}] \\ [0] & [-D_{31}^{\mathrm{III}}] & [D_{11}^{\mathrm{IV}}+D_{33}^{\mathrm{III}}] \end{pmatrix} \begin{pmatrix} \{u_1^{\mathrm{II}}\} \\ \{u_1^{\mathrm{III}}\} \\ \{u_1^{\mathrm{IV}}\} \end{pmatrix} = \begin{pmatrix} \{D_{21}^{\mathrm{I}}u_1^{\mathrm{I}}-D_{12}^{\mathrm{II}}u_2^{\mathrm{II}}\} \\ \{D_{32}^{\mathrm{II}}u_2^{\mathrm{II}}-D_{12}^{\mathrm{III}}u_2^{\mathrm{III}}\} \\ \{D_{32}^{\mathrm{III}}u_2^{\mathrm{III}}-D_{12}^{\mathrm{IV}}u_2^{\mathrm{IV}}\} \end{pmatrix} \tag{13}$$

The unknown particle velocity vector on the domain interface boundaries is obtained by solving Eq. (13).

The sound pressures on S_1^{I} can be found from the first row of Eq. (A1) and Eq. (A2) as

$$\{p_1^{\mathrm{I}}\}=[D_{11}^{\mathrm{I}}]\{u_1^{\mathrm{I}}\}-[D_{12}^{\mathrm{I}}]\{u_1^{\mathrm{II}}\} \tag{14}$$

The sound pressures on S_2^{IV} can be obtained from Eq. (A10) as

$$\{p_2^{\mathrm{IV}}\}=[D_{21}^{\mathrm{IV}}]\{u_1^{\mathrm{IV}}\}+[D_{22}^{\mathrm{IV}}]\{u_2^{\mathrm{IV}}\} \tag{15}$$

Using the averaged sound pressure p_1^{I} and the normal particle velocity u_1^{I} at the inlet of the silencer and the averaged sound pressure p_2^{IV} and *the normal particle velocity* u_2^{IV} *at the outlet, we* can calculate T_{11} and T_{21} as

$$T_{11}=\left.\frac{p_1^{\mathrm{I}}}{p_2^{\mathrm{II}}}\right|_{u_2^{\mathrm{IV}}=0,\ u_1^{\mathrm{I}}=1} \tag{16a}$$

$$T_{21}=\left.\frac{u_1^{\mathrm{I}}}{p_2^{\mathrm{IV}}}\right|_{u_2^{\mathrm{IV}}=0,\ u_1^{\mathrm{I}}=1}=\left.\frac{1}{p_2^{\mathrm{IV}}}\right|_{u_2^{\mathrm{IV}}=0} \tag{16b}$$

Next, T_{12} and T_{22} also can be determined through the similar procedure. At this time, the acoustic equation for the four domain system is reformulated differently from Eq. (13) as follows:

$$\begin{pmatrix} [D_{22}^{\mathrm{I}}+D_{11}^{\mathrm{II}}-Z_{\mathrm{I\,II}}] & [-D_{13}^{\mathrm{II}}] & [0] \\ [-D_{31}^{\mathrm{II}}] & [D_{11}^{\mathrm{III}}+D_{33}^{\mathrm{II}}-Z_{\mathrm{II\,III}}] & [-D_{13}^{\mathrm{III}}] \\ [0] & [-D_{31}^{\mathrm{III}}] & [E_{11}^{\mathrm{IV}}+D_{33}^{\mathrm{III}}] \end{pmatrix} \begin{pmatrix} \{u_1^{\mathrm{II}}\} \\ \{u_1^{\mathrm{III}}\} \\ \{u_1^{\mathrm{IV}}\} \end{pmatrix} = \begin{pmatrix} \{D_{21}^{\mathrm{I}}u_1^{\mathrm{I}}-D_{12}^{\mathrm{II}}u_2^{\mathrm{II}}\} \\ \{D_{32}^{\mathrm{II}}u_2^{\mathrm{II}}-D_{12}^{\mathrm{III}}u_2^{\mathrm{III}}\} \\ \{D_{32}^{\mathrm{III}}u_2^{\mathrm{III}}-E_{12}^{\mathrm{IV}}u_2^{\mathrm{IV}}-E_{13}^{\mathrm{IV}}p_3^{\mathrm{IV}}\} \end{pmatrix} \tag{17}$$

The sound particle velocities on S_2^{IV} can be obtained according to the third row of Eq. (A18) as

$$\{u_3^{IV}\}=[E_{31}^{IV}]\{u_1^{IV}\}+[E_{32}^{IV}]\{u_2^{IV}\}+[E_{33}^{IV}]\{p_3^{IV}\} \quad (18)$$

Using the averaged sound pressure p_1^I and the normal particle velocity u_1^I at the inlet of the silencer and the averaged normal particle velocity u_3^{IV} and the sound pressure p_3^{IV} on the outlet, we compute T_{12} and T_{22} as

$$T_{12}=\frac{p_1^I}{u_3^{II}}\bigg|_{p_3^{I}=0,\ u_1^I=1} \quad (19a)$$

$$T_{22}=\frac{u_1^I}{u_3^{IV}}\bigg|_{p_3^{IV}=0,\ u_1^I=1}=\frac{1}{u_3^{IV}}\bigg|_{p_3^{IV}=0} \quad (16b)$$

The four-pole parameters input into Eq. (12) yield the transmission loss of the silencer.

4. Experiments

4.1 Experimental apparatus and procedure

The schematic diagram of experimental apparatus is shown in Fig. 4. The transmission loss of the air suction silencer is measured by the two-microphone method proposed by Seybert and Ross (Seybert and Ross, 1977). A signal generator gives the specified random-noise signal, which is passed through a power-amplifier before it is fed to horn driver, to create an acoustic field. The signal picked up by each microphone (B&K type 4188) is amplified by a conditioning amplifier, and then goes to a FFT analyzer (B&K PULSE).

To collect and prepare the data for the experimental transmission loss estimation, two microphones are located and spectral densities of signals at the two microphone locations are measured at the inlet and outlet of the silencer.

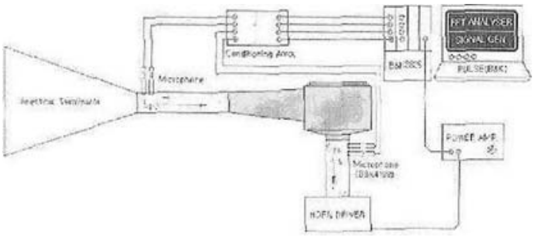

Fig. 4 Experimental setup

4.2 Comparison of analysis results with experimental results

To prove the developed numerical analysis program, first, it is applied to a simple shaped silencer, e.g., a simple expansion silencer. For this acoustic system, the exact analytical solution is available.

The simple circular expansion silencer and the shape parameters of the silencer are shown in Fig. 5 and Table 1, respectively. As shown in Fig. 6, the result of the developed program goes very well with the plane wave theory which is believed to give exact solution under the cutoff frequency, about 1 kHz with the diameter of 0.2 m.

Second application, i.e., the target of this study, is the air suction silencer. The transfer impedance of the air filter with the annular perforated plates with the hole dimeter of 6 mm and the porosity of

Table 1 Dimension of the simple expansion silencer

L	D	d
225 mm	200 mm	50 mm

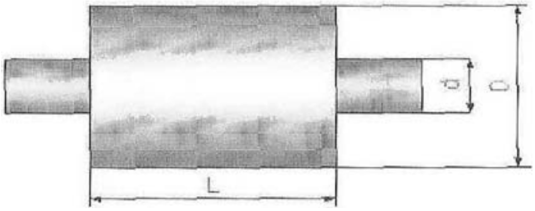

Fig. 5 Simple expansion silencer

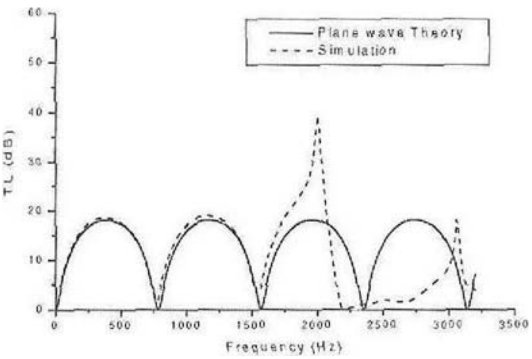

Fig. 6 Comparison of plane wave theory and simulation results for the simple expansion silencer

56% is measured directly by the two-microphone method. Here, we investigated two case trials, in one case where the air filter is removed and the other case where the air filter with the perforated plates is installed. The air suction silencer without the air filter is modeled as shown in Fig. 2, and element size is decided from consideration of the range of analysis frequency and program running time. The transmission loss of the silencer is predicted with the analysis program.

It is shown in Fig. 7 that the analysis of the silencer without the perforated structures predicts so well the experimental result. And also with the perforated structures installed, the developed estimation program approximates reasonably the experimental result as shown in Fig. 8. All these confirmation tests say that the proposed analysis program can be used as a practical means to estimate the sound transmission characteristics of the three dimensional silencers with internal perforated structures.

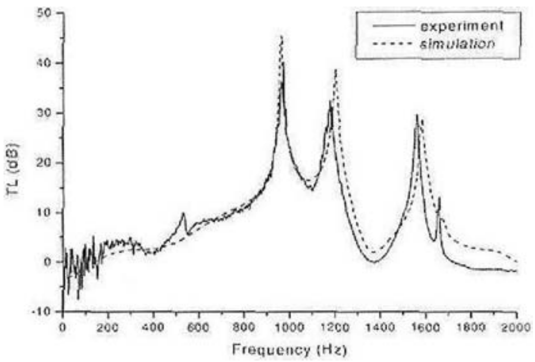

Fig. 7 Transmission losses from experiment and simulation without internal perforated structures

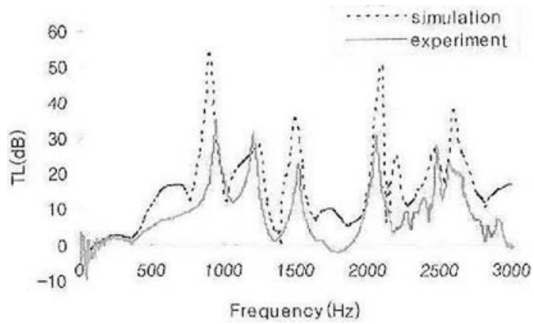

Fig. 8 Transmission losses from experiment and simulation with internal perforated structures

5. Conclusions

We have developed a computationally efficient transmission loss estimation program using multi-domain BEM for three dimensional acoustic structures with perforated internal structures. For computational efficiency, an overall acoustic equation for the entire acoustic structure with the sound particle velocities on the domain boundaries set as the unknowns was formulated and solved to find the acoustic variables which are required to compute the transfer matrix elements to calculate the transmission loss. To validate the performance of the developed program, comparison works with analytic and experimental estimations were tried to a simple expansion silencer and an air suction silencer with/without perforated internal structure. The comparison pairs coincide so reasonably well and so the proposed method can be utilized as a practical means to estimate the sound transmission loss of three dimensional complicated acoustic structures.

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Appendix : Intermediate Equations for Assembling the Overall Acoustic Equation

To determine the transmission loss by using the BEM, the four-pole parameters should be calculated with the inlet and outlet boundary conditions. By putting the first boundary condition so that the normal particle velocity of the inlet of the first domain, Ω_I is 1 and the normal particle velocity of the outlet of fourth domain, Ω_{IV} is 0, T_{11} and T_{21} among the four-pole parameters are calculated. And assuming the secondary boundary condition so that the normal particle velocity of the inlet of first domain, Ω_I is 1 and the sound pressure of the outlet of fourth domain, Ω_{IV} is 0, T_{12} and T_{22} are calculated.

A.1 Intermediate equations for T_{11} nad T_{21}

If the impedance of the interface boundary between the first and second domains is given, the sound pressures and particle velocities can be written as follows:

$$\begin{bmatrix}\{p_1^{I}\}\\ \{p_2^{I}\}\end{bmatrix}=\begin{bmatrix}[D_{11}^{I}] & [D_{12}^{I}]\\ [D_{21}^{I}] & [D_{22}^{I}]\end{bmatrix}\begin{bmatrix}\{u_1^{I}\}\\ \{u_2^{I}\}\end{bmatrix} \tag{A1}$$

$$[D^{I}]=[B^{I}]^{-1}\cdot[A^{I}]$$

$$\{u_1^{II}\}=-\{u_2^{I}\} \tag{A2}$$

$$\{p_1^{II}\}-\{p_2^{I}\}=[Z_{I\,II}]\{u_1^{II}\} \tag{A3}$$

where $[Z_{I\,II}]$ is the impedance matrix of the interface boundary between Ω_I and Ω_{II}. For simplicity, all the parentheses indicating the component vector and matrix for the partial domains are eliminated in Eqs. (A1)-(A3). In this Appendix, we will follow this notation.

Either sound pressures or sound particle velocities are known at every point on the boundary S_2^{II}. Sound pressures and sound particle velocities are unknown but acoustic impedance is given on the boundaries S_1^{II} and S_3^{II}. In the case of Ω_{II}, Eq. (A1) can be replaced as follows:

$$\begin{bmatrix}\{p_1^{II}\}\\ \{p_2^{II}\}\\ \{p_3^{II}\}\end{bmatrix}=\begin{bmatrix}[D_{11}^{II}] & [D_{12}^{II}] & [D_{13}^{II}]\\ [D_{21}^{II}] & [D_{22}^{II}] & [D_{23}^{II}]\\ [D_{31}^{II}] & [D_{32}^{II}] & [D_{33}^{II}]\end{bmatrix}\begin{bmatrix}\{u_1^{II}\}\\ \{u_2^{II}\}\\ \{u_3^{II}\}\end{bmatrix} \tag{A4}$$

$$[D^{II}]=[B^{II}]^{-1}\cdot[A^{II}]$$

$$\{u_1^{III}\}=-\{u_3^{II}\} \tag{A5}$$

$$\{p_1^{III}\}-\{p_3^{II}\}=[Z_{II\,III}]\{u_1^{III}\} \tag{A6}$$

where $[Z_{II\,III}]$ is the impedance matrix of the interface boundary between Ω_{II} and Ω_{III}.

In, Ω_{III} same equation as Eq. (A4) holds except superscript "II" is replaced by "III".

$$\{p_1^{IV}\}=\{p_3^{III}\} \tag{A7}$$

$$\{u_1^{IV}\}=-\{u_3^{III}\} \tag{A8}$$

In, Ω_{IV} just the same as Ω_I, Eq. (A1) can be explained as follows:

$$\begin{bmatrix}\{p_1^{IV}\}\\ \{p_2^{IV}\}\end{bmatrix}=\begin{bmatrix}[D_{11}^{IV}] & [D_{12}^{IV}]\\ [D_{21}^{IV}] & [D_{22}^{IV}]\end{bmatrix}\begin{bmatrix}\{u_1^{IV}\}\\ \{u_2^{IV}\}\end{bmatrix} \tag{A9}$$

$$[D^{IV}]=[B^{IV}]^{-1}\cdot[A^{IV}]$$

The second row of Eq. (A1) with Eq. (A2) yields the following relation.

$$\{p_2^{I}\}=[D_{21}^{I}]\{u_1^{I}\}-[D_{22}^{I}]\{u_1^{II}\} \tag{A10}$$

Eq. (A3) with Eq. (A10), Eq. (A5) and the first row of Eq. (A4) leads to

$$\begin{aligned}&([D^{\mathrm{I}}_{22}]+[D^{\mathrm{II}}_{11}]+[Z_{\mathrm{I\,II}}])+\{u^{\mathrm{II}}_{1}\}-[D^{\mathrm{II}}_{13}]\{u^{\mathrm{III}}_{1}\}\\&=[D^{\mathrm{I}}_{21}]\{u^{\mathrm{I}}_{1}\}-[D^{\mathrm{II}}_{12}]\{u^{\mathrm{II}}_{2}\}\end{aligned} \tag{A11}$$

The third row of Eq. (A4) with Eq. (A5) results the equation

$$\{p^{\mathrm{II}}_{3}\}=[D^{\mathrm{II}}_{31}]\{u^{\mathrm{II}}_{1}\}+[D^{\mathrm{II}}_{32}]\{u^{\mathrm{II}}_{2}\}-[D^{\mathrm{II}}_{33}]\{u^{\mathrm{III}}_{1}\} \tag{A12}$$

The first row of Eq. (A4) in which "II" is replaced by "III" with Eq. (A8) constructs the following equation.

$$\{p^{\mathrm{III}}_{1}\}=[D^{\mathrm{III}}_{11}]\{u^{\mathrm{III}}_{1}\}+[D^{\mathrm{III}}_{12}]\{u^{\mathrm{III}}_{2}\}-[D^{\mathrm{III}}_{12}]\{u^{\mathrm{IV}}_{1}\} \tag{A13}$$

Eq. (A6) with Eq. (A12) and Eq. (A13) makes the equation

$$\begin{aligned}&-[D^{\mathrm{II}}_{31}]\{u^{\mathrm{II}}_{1}\}+([D^{\mathrm{III}}_{11}]+[D^{\mathrm{II}}_{33}]-[Z_{\mathrm{II\,III}}])\{u^{\mathrm{III}}_{1}\}-[D^{\mathrm{III}}_{13}]\{u^{\mathrm{IV}}_{1}\}\\&=[D^{\mathrm{II}}_{32}]\{u^{\mathrm{II}}_{2}\}-[D^{\mathrm{III}}_{12}]\{u^{\mathrm{III}}_{2}\}\end{aligned} \tag{A14}$$

The third row of Eq. (A4) in which "II" is replaced by "III" with Eq. (A8) leads to

$$\{p^{\mathrm{III}}_{3}\}=[D^{\mathrm{III}}_{31}]\{u^{\mathrm{III}}_{1}\}+[D^{\mathrm{III}}_{32}]\{u^{\mathrm{III}}_{2}\}-[D^{\mathrm{III}}_{33}]\{u^{\mathrm{IV}}_{1}\} \tag{A15}$$

Eq. (A7) with the first row of Eq. (A9) and Eq. (A15) gives the following equation.

$$\begin{aligned}&([D^{\mathrm{IV}}_{11}]+[D^{\mathrm{III}}_{33}])\{u^{\mathrm{IV}}_{1}\}-[D^{\mathrm{III}}_{31}]\{u^{\mathrm{III}}_{1}\}\\&=[D^{\mathrm{III}}_{32}]\{u^{\mathrm{III}}_{2}\}-[D^{\mathrm{IV}}_{12}]\{u^{\mathrm{IV}}_{2}\}\end{aligned} \tag{A16}$$

Eq. (A11), Eq. (A14), and Eq. (A16) are reassembled to formulate an overall equation for the unknown particle velocities on the domain interface boundaries as Eq. (13).

A.2 Intermediate equations for T_{12} and T_{22}

If T_{12} and T_{22} among the four-pole parameters are calculated, Eq. (A9) can be expressed as follows:

$$\begin{bmatrix}\{p^{\mathrm{IV}}_{1}\}\\\{p^{\mathrm{IV}}_{2}\}\\\{p^{\mathrm{IV}}_{3}\}\end{bmatrix}=\begin{bmatrix}[E^{\mathrm{IV}}_{11}]&[E^{\mathrm{IV}}_{12}]&[E^{\mathrm{IV}}_{13}]\\{}[E^{\mathrm{IV}}_{21}]&[E^{\mathrm{IV}}_{22}]&[E^{\mathrm{IV}}_{23}]\\{}[E^{\mathrm{IV}}_{31}]&[E^{\mathrm{IV}}_{32}]&[E^{\mathrm{IV}}_{33}]\end{bmatrix}\begin{bmatrix}\{u^{\mathrm{IV}}_{1}\}\\\{u^{\mathrm{IV}}_{2}\}\\\{u^{\mathrm{IV}}_{1}\}\end{bmatrix} \tag{A17}$$

$$[E^{\mathrm{IV}}]=[B^{\mathrm{IV}}]^{-1}\cdot[A^{\mathrm{IV}}]$$

Eq. (A7) is combined with the third row of Eq. (A4) in which "II" is replaced by "III" and the first row of Eq. (A17) to yield the following equation.

$$\begin{aligned}&-[D^{\mathrm{III}}_{31}]\{u^{\mathrm{III}}_{1}\}+([E^{\mathrm{IV}}_{11}]+[D^{\mathrm{III}}_{33}])\{u^{\mathrm{IV}}_{1}\}\\&=[D^{\mathrm{III}}_{32}]\{u^{\mathrm{III}}_{2}\}-[E^{\mathrm{IV}}_{12}]\{u^{\mathrm{IV}}_{2}\}-[E^{\mathrm{IV}}_{13}]\{p^{\mathrm{IV}}_{3}\}\end{aligned} \tag{A18}$$

Eq. (A11), Eq. (A14) and Eq. (A18) are reassembled to formulate an overall equation for the unknown particle velocities on the domain interface boundaries as Eq. (17).